Generation of cyclically 4-connected cubic graphs

Roman Nedela

In my talk I prove that the class of cyclically 4-connected cubic graphs can be generated from three small graphs, the complete bipartite graph $K_{3,3}$, the cube and the twisted cube, by means of two locally defined operations. First one is the reverse of vertex-reduction and the second is 4-reduction.

The vertex-reduction of G is defined by removal a vertex v and smoothing the 2-valent vertices of G - v.

The 4-reduction removes a 4-edge cut C separating a quadrangle Q, takes H = G - Q and restores the cubicity by adding two edges. The statement can be used in inductive proofs.

The core of the proof consists in an argument that one can always find a proper vertex v, or a proper quadrangle Q, so that applying the vertex reduction with respect to v, or 4-reduction with respect to Q, the obtained cubic graph remains cyclically 4-connected.